Is Relativistic Quantum Mechanics Compatible with Special Relativity?

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The transformation from a time-dependent random walk to quantum mechanics converts a modified Bessel function into an ordinary one together with a phase factor $e^{i\pi/2}$ for each time the electron flips both direction and handedness. Causality requires the argument to be greater than the order of the Bessel function. Assuming equal probabilities for jumps ± 1, the normalized modified Bessel function of an imaginary argument is the solution of the finite difference differential Schrödinger equation whereas the same function of a real argument satisfies the diffusion equation. In the nonrelativistic limit, the stability condition of the difference scheme contains the mass whereas in the ultrarelativistic limit only the velocity of light appears. Particle waves in the nonrelativistic limit become elastic waves in the ultrarelativistic limit with a phase shift in the frequency and wave number of $\pi/2$. The ordinary Bessel function satisfies a second order recurrence relation which is a finite difference differential wave equation, using non-nearest neighbors, whose solutions are the chirality components of a free-particle in the zero fermion mass limit. Reintroducing the mass by a phase transformation transforms the wave equation into the Klein-Gordon equation but does not admit a solution in terms of ordinary Bessel functions. However, a sign change of the mass term permits a solution in terms of a modified Bessel function whose recurrence formulas produce all the results of special relativity. The Lorentz transformation maximizes the integral of the modified Bessel function and determines the paths of steepest descent in the classical limit. If the definitions of frequency and wave number in terms of the phase were used in special relativity, the condition that the frame be inertial would equate the superluminal phase velocity with the particle velocity in violation of causality. In order to get surfaces of constant phase to move at the group velocity, an integrating factor is required which determines how the intensity decays in time. The phase correlation between neighboring sites in quantum mechanics is given by the phase factor for the electron to reverse its direction, whereas, in special relativity, it is given by the Doppler shift.

Key words: Random Walks; Quantum Mechanics; Special Relativity; Ordinary and Modified Bessel Functions.